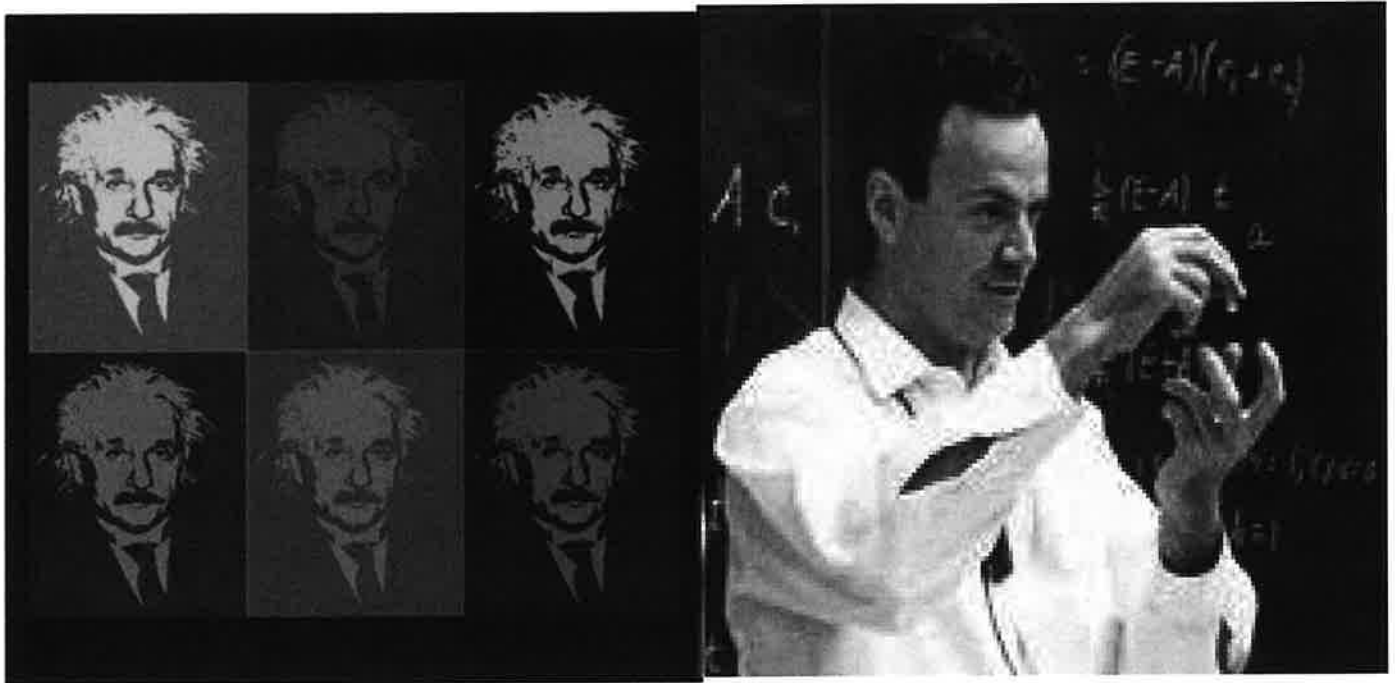


Introductory Maths Unit for Year 12 Physics



Contents

Chapter	Topic
1.	Introduction
2.	Standard Form
3.	Prefixes/Converting Unit Magnitudes
4.	Re-arranging Equations
5.	Significant Figures
6.	Using Your Calculator
7.	Extension : Converting Units to SI Units



Chapter 1: Introduction

The purpose of this introductory unit is to help you develop the core skills needed to solve numerical problems which will make your Year 12 Physics studies much more enjoyable and successful than they otherwise would be. Without these core skills solving problems becomes much more difficult if not impossible, a bit like trying to build a house with no wood or bricks. A bit of work before the course starts will pay huge dividends later and allow you to work and learn much more efficiently.

The core maths skills you will need are:

- *Using a Calculator*
- *Significant figures*
- *Standard form*
- *Unit prefixes*
- *Powers of units (converting units)*
- *Rearranging Algebra*

How to answer questions

The key to success is to break numerical problems, where calculations are necessary, into smaller, simpler steps which can be followed every time.

The steps can be summarised as follows:-

Step 1: *Write down the values of everything you are given and put a question mark next to what you are asked to work out.*

Step 2: *Convert all the values into SI units i.e. time in seconds, distances in metres and so on.*

Step 3: *Pick an equation that contains the values we know and the quantity we are trying to work out.*

Step 4: *Re-arrange the equation so what we are trying to work out is the subject.*

Step 5: *Insert the values into the equation including the units.*

Step 6: *Type it into our calculator to get the answer and quote the answer to a reasonable number of significant figures and with units.*

Step 7: *Pause for one moment and think about if our answer is sensible.*

With experience some of these steps can be done more quickly or in your head but **you should always show your working**. This is for several reasons:-

1. If you don't show your working, you will needlessly lose many marks in the exam (probably enough to drop your score by one whole grade, i.e. from B → C).
2. It will help make the steps outlined above more apparent and easy to follow when tackling numerical problems.
3. It will help you see where you have made mistakes when you check your answers
4. It makes it easier for the teacher to see where you have gone wrong and therefore help you learn more quickly and effectively.

Chapter 2: Standard Form

Why use standard form? Standard form is used to make very large or very small numbers easier to read. Standard form also makes it easier to put large or small numbers in order of size.

In Physics, we often deal with quantities that are either really large, such as a parsec

$$1 \text{ pc} = 30,900,000,000,000,000 \text{ m}$$

Or really small like Planck's Constant:-

$$h = 0.00000000000000000000000000000000000663 \text{ Js}$$

Now, it would be tiresome to write out numbers like this over and over again and so we use a different notation known as standard form. Standard form shows the magnitude (size) of the number as powers of ten. We write a number between 1 and 10 and then show it multiplied by a power of 10. The number of powers of ten is called the index.

For example

$$1.234 \times 10^4$$

$$1.234 \times 10^{-4}$$

This means $1.234 \times (10 \times 10 \times 10 \times 10)$
Or move DP 4 places to the right

$1.234 \times (1 \div 10 \div 10 \div 10 \div 10)$
or move DP 4 places to the left

Which is **12340**

0.0001234

Let's see some more examples.

$$0.523 = 5.23 \times 10^{-1} \quad (\text{note that } \times 10^{-1} \text{ means divide } 5.23 \text{ by } 10)$$

$$52.3 = 5.23 \times 10^1 \quad (\text{note that } \times 10^1 \text{ means multiply } 5.23 \text{ by } 10)$$

$$523 = 5.23 \times 10^2 \quad (\text{note that } \times 10^2 \text{ means multiply } 5.23 \text{ by } 100)$$

$$5230 = 5.23 \times 10^3 \quad (\text{note that } \times 10^3 \text{ means multiply } 5.23 \text{ by } 1000)$$

$$0.00523 = 5.23 \times 10^{-3} \quad (\text{note that } \times 10^{-3} \text{ means divide } 5.23 \text{ by } 1000)$$

Note that the sign (positive or negative) in the index tells you whether you are dividing or multiplying; a positive number means you are multiplying (getting bigger) and a negative number means you are dividing (getting smaller). The number tells you how many times you are either dividing or multiplying by 10. So 1.60×10^{-19} means take the number 1.60 and divide it by 10 nineteen times (divide by 10^{19}) i.e. move the decimal point 19 places to the left.

VIDEO CLIP: <https://www.youtube.com/watch?v=3OMAAOpMj00>

Check out: There is a video on youtube showing Mr McDonald explaining how to convert numbers into standard form and back again. Just search for "**you tube ronan mcdonald standard form**"

And to go back to our examples :-

$$1 \text{ pc} = 3.09 \times 10^{16} \text{ m}$$

$$h = 6.63 \times 10^{-34} \text{ Js}$$

So this is a much shorter way of writing these numbers!

To put a list of large numbers in order is difficult because it takes time to count the number of digits and hence determine the magnitude of the number.

Exercise on Chapter 2: Standard Form

1. Put these numbers in order of size,

5239824 , 25634897 , 5682147 , 86351473 , 1258964755 142586479, 648523154

But it is easier to order large numbers when they are written in standard form.

2. Put these numbers in order of size,

5.239×10^6 , 2.563×10^7 , 5.682×10^6 , 8.635×10^7 , 1.258×10^9 1.425×10^8 , 6.485×10^8

You can see that it is easier to work with large numbers written in standard form. To do this we must be able to convert from one form into the other.

3. Convert these numbers into normal form.

a) 5.239×10^3

b) 4.543×10^4

c) 9.382×10^2

d) 6.665×10^6

4. Convert these numbers into standard form.

a) 65345 (how many times do you multiply 6.5345 by 10 to get 65345 ?)

b) 28748

c) 548454

d) 486856

Standard form can also be used to write small numbers

e.g. $0.00056 = 5.6 \times 10^{-4}$

5. Convert these numbers into normal form.

a) 8.34×10^{-3}

b) 2.541×10^{-8}

c) 1.01×10^{-5}

6. Convert these numbers to standard form.

a) 0.000567

b) 0.987

c) 0.0000605

Rules when using powers of ten in calculations

To multiply powers of ten, ADD THE INDICES. eg, $10^5 \times 10^3 = 10^8$

To divide powers of ten, SUBTRACT THE INDICES. eg, $10^5 \div 10^3 = 10^2$

7. Calculate, giving answers in standard form,

a) $(3.45 \times 10^{-5} + 9.5 \times 10^{-6}) \div 0.0024$

b) $2.31 \times 10^5 \times 3.98 \times 10^{-3} + 0.0013$

Order of Magnitude

If a number is rounded up to the nearest power of ten, we say we are giving an order of magnitude value.

The average separation of the galaxies is ~ 106 light years. The symbol \sim is used to mean 'to within an order of magnitude'.

The wavelength of red light is 7×10^{-7} m and of violet light is 4×10^{-7} m. So the average wavelength is $\sim 10^{-7}$ m.

Chapter 3: Prefixes & Converting Unit Magnitudes

How to use and convert prefixes

Often in Physics, quantities are written using prefixes which is an even shorter way of writing numbers than standard form. For example instead of writing 2.95×10^{-9} m we can write 2.95 nm where n means nano and is a short way of writing $\times 10^{-9}$.

Search the internet to find the information to fill in the missing parts of the table below.

Prefix	Symbol	Name	Multiplier
femto	f	quadrillionth	10^{-15}
pico	p	trillionth	1
nano	n	billionth	
micro		millionth	
milli	m	thousandth	
kilo	k	thousand	10^3
mega		million	
giga	G	billion [†]	
tera		trillion [†]	
peta	P	quadrillion	10^{15}

Again, it is essential you know all of these to ensure that you don't lose easy marks when answering numerical problems.

Other Prefixes you may see are:

centi	c	hundredth	10^{-2}
deci	d	tenth	10^{-1}
hecto	h	hundred	10^2
deka	da	ten	10^1

When you are given a variable with a prefix you must convert it into its numerical equivalent in standard form before you use it in an equation.

FOLLOW THIS! Always start by replacing the prefix symbol with its equivalent multiplier.

For example: $0.16 \mu\text{A} = 0.16 \times 10^{-6} \text{ A} = 0.00000016\text{A}$

$3 \text{ km} = 3000\text{m} = 3 \times 10^3 \text{ m}$

$10 \text{ ns} = 10 \times 10^{-9} \text{ s} = 0.00000001 \text{ s}$

DO NOT get tempted to follow this further (for example: $0.16 \times 10^{-6} \text{ A} = 1.6 \times 10^{-7} \text{ A}$ and also $10 \times 10^{-9} \text{ s} = 10^{-8} \text{ s}$) unless you are absolutely confident that you will do it correctly. It is always safer to stop at the first step ($10 \times 10^{-9} \text{ s}$) and type it like this into your calculator.

VIDEO CLIP: <https://www.youtube.com/watch?v=54tag7XQFpw>

Check out: There is a video on youtube showing Mr McDonald explaining how to convert between different prefixes. Just search for “you tube alom shaha prefixes used with base units”

NOW TRY THIS!

Exercise on Chapter 3: Prefixes/Converting Unit Magnitudes

1.4 kW =

10 μ C =

24 cm =

340 MW =

46 pF =

0.03 mA =

52 Gbytes =

43 k Ω =

Converting between unit magnitudes for distances.

Convert the following: (Remember that milli = 10^{-3} and centi = 10^{-2})

1. 5.46m to cm =
2. 65mm to m =
3. 3cm to m =
4. 0.98m to mm =
5. 34cm to mm =
6. 76mm to cm =

Converting between unit magnitudes for areas and volumes

It's really important that when we convert areas and volumes that we don't forget to square or cube the unit.

Example

Let's take the example of converting a sugar cube of volume 1 cm³ into m³.

If we just use the normal conversion, then 1 cm³ = 1 x 10⁻² m³ ← **Wrong Answer!**

STOP! Let's think about this one second:

Imagine in your head a box 1m by 1m by 1m, how many sugar cubes could you fit in there? A **lot** more than 100! That would only fill up one line along one of the bottom edges of the box! **So our answer must be wrong.**

What we have to do is do the conversion and then cube it, like this:-

$$1 \text{ cm}^3 = 1 (\text{x } 10^{-2} \text{ m})^3 = 1 \text{ x } 10^{-6} \text{ m}^3.$$

So this means we could fit a million sugar cubes in the box, which is right.

Exercise - Converting Areas and Volumes

1. What is 5.2 mm^3 in m^3 ? =
2. What is 24cm^2 in m^2 ? =
3. What is 34 m^3 in μm^3 ? =
4. What is $0.96 \times 10^6 \text{ m}^2$ in km^2 ? =
5. Convert 34 Mm^3 into pm^3 . =

Chapter 4: Re-arranging Equations

The first step in learning to manipulate an equation is your ability to see how it is done once and then repeat the process again and again until it becomes second nature to you.

In order to show the process once I will be using letters rather than physical concepts.

You can rearrange an equation $a = b \times c$ with

b as the subject $b = \frac{a}{c}$

or c as the subject $c = \frac{a}{b}$

Any of these three symbols a , b , or c can be itself a summation, a subtraction, a multiplication, a division, or a combination of all. So, when you see a more complicated equation, try to identify its three individual parts a, b, c before you start rearranging it.

Worked examples

Equation	First Rearrangement	Second Rearrangement
$v = f \times \lambda$	$f = \frac{v}{\lambda}$	$\lambda = \frac{v}{f}$
$T = \frac{1}{f}$	$1 = T \times f$	$f = \frac{1}{T}$
$\frac{1}{v} = \frac{1}{u} + \frac{1}{f}$	$1 = v \times \left(\frac{1}{u} + \frac{1}{f} \right)$	$v = \frac{1}{\frac{1}{u} + \frac{1}{f}}$

NOW TRY THIS!

From now on the multiplication sign will not be shown, so $a = b \times c$ will be simply written as $a = bc$

The delta sign (Δ) means change in.

Equation	First Rearrangement	Second Rearrangement
(Power of lens) $P = \frac{1}{f}$	$1 =$	$f =$
(Magnification of lens) $m = \frac{v}{u}$	$v =$	$u =$
(refractive index) $n = \frac{c}{v}$	$c =$	$v =$
(current) $I = \frac{\Delta Q}{\Delta t}$		
(power) $P = I^2 R$		
(power) $P = \frac{V^2}{R}$		

Further Rearranging Practice

1. $a = bc$ $b =$
2. $a = \frac{b}{c}$ $b =$ $c =$
3. $a = b - c$ $c =$
4. $a = b + c$ $b =$
5. $a = \frac{b}{c} + d$ $c =$
6. $a = \frac{b}{c} - d$ $c =$
7. $a = \frac{bc}{d}$ $d =$ $b =$
8. $a = \frac{(b + c)}{d}$ $c =$
9. $a = \frac{b}{c} + \frac{d}{e}$ $e =$

Chapter 5: Significant Figures

When you use a calculator to work out a numerical answer you know that this often results in a large number of decimal places and in most cases the final few digits are “not significant”.

Numbers in physics are not just numbers, they represent **physical quantities** that have been **measured or calculated** from measurements. With practical data, the precision of the measuring instrument (and the uncertainty in your measurement) affects how many figures will be significant. It is important to record your data and your answers to calculations to a reasonable number of significant figures. Too many and your answer is claiming an accuracy that it does not have. Too few and you are not showing the precision required in scientific analysis.

The Rules

1. All non-zero digits are significant.
2. In a number without a decimal point, only zeros BETWEEN non-zero digits are significant. E.g. significant in 12001 but not in 12100
3. In a number with a decimal point, all zeros to the right of the right-most non-zero digit are significant. 12.100 → 5 s.f.

Ambiguous significant figures.

Be careful with trailing zeros. 230m could have 2 or 3 sig figs depending on the precision of the measuring instrument. It is better to use standard form to show whether it is 2.3×10^2 m or 2.30×10^2 m.

Examples

39.389 → 5 s.f.

1200000000000000 → 2 s.f

3400.000 → 7 s.f.

34224000 → 5 s.f.

200000.0004 → 10 s.f.

HOW MANY SIGNIFICANT FIGURES TO USE

For practical data, you should be guided by the **RESOLUTION** of your measurements.

For calculations **YOU MUST** use the same number of figures as the data in the question with the **LOWEST NUMBER** of significant figures.

Exercise:-

1. How many significant figures are there in these numbers?

- a) 609W
- b) 3.4kg
- c) 21.67m
- d) 400.0N
- e) 10.01s
- f) 5MW
- g) 6.0s
- h) 9.8ms^{-2}
- i) $3.0 \times 10^8\text{ms}^{-1}$

2. Write these measurements to two significant figures.

- a) 19.47m
- b) 115 km
- c) 21.0 s
- d) 6.63×10^{34} Js
- e) 1.673×10^{-27} kg
- f) 5s
- g) Use the equation $V = IR$ to calculate the electric current I through a 3300Ω resistor when the potential difference (V) is 12V

Chapter 6: Using Your Calculator

Quick Exercise

- Using your calculator, Evaluate:-

$$\frac{30}{5 \times 3} = ?$$

What answer did you get?

Work it out in your head - Is your answer correct?

Order of Operations

Your calculator has a rule to decide which operation to do first which is summarised by the word BODMAS, which stands for the order in which operations are done:-

1. B - Brackets first
2. O - Orders (i.e. Powers and Square Roots, etc.)
3. DM - Division and Multiplication (left-to-right)
4. AS - Addition and Subtraction (left-to-right)

So if we type in the numbers like this:-

$30 \div 5 \times 3 = 6 \times 3 = 18$ ← Left to Right is the conventional order and is what your calculator does.

But if we use brackets we can get the right answer:- $30 \div (5 \times 3) = 30 \div 15 = 2$

Note that the fact that the 5 and 3 are put on the bottom implies they should be multiplied first. You will need to be able to use your calculator correctly and be familiar with scientific notation, such as standard form, brackets etc.

e.g. $3\,670\,000 = 3.67 \times 10^6$

$$0.0000367 = 3.67 \times 10^{-4}$$

To enter 3.67×10^6 into your calculator press:

$$3.67 \text{ exp } 6$$

or use the $\times 10^x$ button



Note that 10^8 means 1×10^8 and so must be keyed in as $1 \text{ exp } 8$ not $10 \text{ exp } 8$!

As a result when I write out what I know I write out 1×10^8 to remind myself to do this.

Exercise In each case, find the value of "y".

1. $y = (7.5 \times 10^3)^2$

2. $y = \frac{(1.3 \times 10^3) \times (1.6 \times 10^{-4})}{(6.6 \times 10^6) + (3.27 \times 10^{-3})}$

3. $y = \frac{(5.6 \times 10^{-4})^2 \times (7.8 \times 10^8)}{(6.6 \times 10^{-11}) \times (9.1 \times 10^{-2})^2}$

4. $y = \frac{(4.12 \times 10^3) + (6.5 \times 10^2)}{(2.3 \times 10^4) \times (8.1 \times 10^2)}$

Chapter 7: Physical Quantities/Units

Naked Numbers

Without units, numbers are meaningless.

1. Use the internet to find out what happened to the Mars Climate Observer spacecraft in 1999 and why. Write down what you found out below.

You must always write down units with your answer. In exams, **you lose the last calculation mark (for the correct answer!!!) if you have a unit error.**

A table of quantities with their units is shown below along with the most commonly used symbols for both the quantities and units.

Note that in GCSE we wrote units like metres per second in the format of m/s but in A-level it is written as ms^{-1} , and this is the standard way units are written at university level Physics.

A Table of Quantities with Units

Quantity	Quantity Symbol	SI Unit	Unit Symbol
Length	L or l	Metre	m
Distance	s	Metre	m
Height	h	Metre	m
Thickness (of a Wire)	d	Metre	m
Wavelength	λ	Metre	m
Mass	m or M	Kilogram	kg
Time	t	Second	s
Period	T	Second	s
Temperature	T	Kelvin	K
Current	I	Ampere	A
Potential Difference	V	Volt	V
Area	A	Metres squared	m^2

Quantity	Quantity Symbol	SI Unit	Unit Symbol
Volume	V	Metres cubed	m ³
Density	ρ	Kilograms per metre cubed	kg m ⁻³
Force	F	Newton	N
Initial Velocity	u	Metres per second	ms ⁻¹
Final Velocity	v	Metres per second	ms ⁻¹
Energy	E	Joule	J
Kinetic Energy	E _k	Joule	J
Work Done	W	Joule	J
Power	P	Watt	W
Luminosity	L	Watt	W
Frequency	f	Hertz	Hz
Charge	Q	Coulomb	C
Resistance	R	Ohm	Ω
Electromotive Force	ϵ	Volt	V
Resistivity	ρ	Ohm Metre	Ωm
Work Function	ϕ	Joule	J
Momentum	p	kilogram metres per second	kg ms ⁻¹
Specific Charge	q	Coulombs per kilogram	C kg ⁻¹
Planck's Constant	h	Joule seconds	Js
Gravitational Field Strength	g	Newtons per kilogram	N kg ⁻¹

Highlight all the quantities you recognise